A problem of Diophantus and Dickson's conjecture

Andrej Dujella

Abstract. A Diophantine m-tuple with the property $D(n)$, where n is an integer, is defined as a set of m positive integers with the property that the product of its any two distinct elements increased by n is a perfect square. It is known that if n is of the form $4k+2$, then there does not exist a Diophantine quadruple with the property $D(n)$. The author has formerly proved that if n is not of the form $4k+2$ and $n \notin \{-15, -12, -7, -4, -3, -1, 3, 5, 7, 8, 12, 13, 15, 20, 21, 24, 28, 32, 48, 60, 84\}$, then there exist at least two distinct Diophantine quadruples with the property $D(n)$.

The main problem of this paper is to consider the set U of all integers n, not of the form $4k+2$, such that there exist at most two distinct Diophantine quadruples with the property $D(n)$. One open question is whether the set U is finite or not. It can be proved that if $n \in U$ and $|n| > 48$, then n can be represented in one of the following forms: $4k+3$, $16k+12$, $8k+5$, $32k+20$. The main results of the this paper are:

If $n \in U \setminus \{-9, -1, 3, 7, 11\}$ and $n \equiv 3 \pmod 4$, then the integers $|n-1|/2$, $|n-9|/2$ and $|9n-1|/2$ are primes, and either $|n|$ is prime or n is the product of twin primes.

If $n \in U \setminus \{-27, -3, 5, 13, 21, 45\}$ and $n \equiv 5 \pmod 8$, then the integers $|n|$, $|n-1|/4$, $|n-9|/4$ and $|9n-1|/4$ are primes.

1991 Mathematics Subject Classification: 11D09, 11A41

1. Introduction

The Greek mathematician Diophantus of Alexandria noted that the rational numbers $\frac{1}{16}$, $\frac{33}{16}$, $\frac{17}{4}$ and $\frac{105}{16}$ have the following property: the product of any two of them increased by 1 is a square of a rational number (see [3]). The first set of four integers with the above property was found by Fermat, and it was $\{1, 3, 8, 120\}$. In 1969, Davenport and Baker [1] showed that if d is a positive integer such that the set $\{1, 3, 8, d\}$ has the property of Diophantus, then d has to be 120.

Let n be an integer. A set of positive integers $\{a_1, a_2, \ldots, a_m\}$ is said to have *the property* $D(n)$, if for all $1 \le i < j \le m$ the following holds: $a_i a_j + n = b_{ij}^2$, where b_{ij} is an integer. Such a set is called *a Diophantine m-tuple.* If n is an integer of the form $4k+2$, $k \in \mathbf{Z}$, then there does not exist Diophantine quadruple with the property $D(n)$ (see [2, Theorem 1], [4, Theorem 4] or [9, p. 802]). If an integer n is not of the form $4k+2$ and $n \notin \{-15,-12,-7,-4,-3,-1,3,5,7,8,12,13,15,20,21,24, 28,32,48,60,84\}$, then there exist at least two distinct Diophantine quadruples with the property $D(n)$ (see [4, Theorems 5 and 6] and [5, p. 315]). The proof of the former result is based on the fact that the sets

$$\{m, m(3k+1)^2+2k, m(3k+2)^2+2k+2, 9m(2k+1)^2+8k+4\},$$

$$\{m, mk^2-2k-2, m(k+1)^2-2k, m(2k+1)^2-8k-4\}$$

have the property $D(2(2k+1)m+1)$. These formulas are used in [7] and the above results are generalized to the set of Gaussian integers. More formulas of this type were obtained in [6].

These formulas were used in [8], where some improvements of the results of [4] were obtained. It was proved that if $n \equiv 1 \pmod 8$ and $n \notin \{-15,-7,17,33\}$, or $n \equiv 4 \pmod{32}$ and $n \notin \{-28,68\}$, or $n \equiv 0 \pmod{16}$ and $n \notin \{-16,32,48, 80\}$, then there exist at least six, and if $n \equiv 8 \pmod{16}$ and $n \notin \{-8,8,24,40\}$, then there exist at least four distinct Diophantine quadruples with the property $D(n)$. These results imply that if an integer n is not of the form $4k+2$, $|n| > 48$, and there exist at most two distinct Diophantine quadruples with the property $D(n)$, then n can be represented in one of the following forms:

$$4k+3, \qquad 16k+12, \qquad 8k+5, \qquad 32k+20.$$

The main problem of this paper is to consider those n for which *there are at most two Diophantine quadruples with the property* $D(n)$. We will prove that for an integer n, not of the from $4k+2$, the assumption that there exist at most two distinct Diophantine quadruples with the property $D(n)$ has very strong consequences, which are connected with the problem of existence of primes in arithmetical progressions.

Since multiplying all elements of quadruples with the properties $D(4k+3)$ and $D(8k+5)$ by 2 we obtain the quadruples with the properties $D(16k+12)$ and $D(32k+20)$, respectively (by [4, Remark 3], all quadruples with the property $D(16k+12)$ can be obtained on this way), we will restrict our attention to the integers of the forms $4k+3$ and $8k+5$.

2. The case n=4k+3

Theorem 1. *Let n be an integer such that $n \equiv 3 \pmod 4$, $n \notin \{-9,-1,3,7,11\}$, and there exist at most two distinct Diophantine quadruples with the property $D(n)$. Then the integers $|n-1|/2$, $|n-9|/2$ and $|9n-1|/2$ are primes. Furthermore, either the integer $|n|$ is prime or $n = pq$, where p and q are twin primes.*

To prove this theorem we need the following lemmas.

Lemma 1. *Let n be an integer such that $n \equiv 3 \pmod 4$ and let $n = st$, where s and t are integers such that $s \geq 1$ and $s-t > 2$. Let $v = (s-t-2)/4$. Then the* *set*

$$\{1,(3v+1)^2+2vt,(3v+2)^2+2(v+1)t,9(2v+1)^2+4(2v+1)t\} \tag{2.1}$$

is a Diophantine quadruple with the property $D(n)$*.*

Proof. From $st\equiv 3 \pmod 4$ it follows that $s\equiv t+2 \pmod 4$. Hence v is a positive integer. Set

$$\begin{aligned} b &= (3v+1)^2+2vt \\ c &= (3v+2)^2+2(v+1)t \\ d &= 9(2v+1)^2+4(2v+1)t. \end{aligned}$$

By [6, proof of Theorem 1], the product of any two distinct elements of the set $\{1,b,c,d\}$ increased by n is a perfect square. Thus, it is sufficient to prove that 1, b, c and d are distinct positive integers. We have:

$$\begin{aligned} b-1 &= v(9v+2t+6)=v(v+2s+2)>0 \\ c-1 &= (v+1)(9v+2t+3)=(v+1)(v+2s-1)>0 \\ d-1 &= (2v+1)(18v+4t+9)-1=(2v+1)(2v+4s+1)-1>0 \\ c-b &= 6v+2t+3=(3s+t)/2\neq 0 \\ d-b &= (3v+2)(9v+2t+4)=(3v+2)(v+2s)>0 \\ d-c &= (3v+1)(9v+2t+5)=(3v+1)(v+2s+1)>0, \end{aligned}$$

which proves the lemma. □

Lemma 2. *If the integer* $|2k+1|$ *is composite, then there exists a Diophantine quadruple* $\{a,b,c,d\}\subset \mathbf{N}\setminus\{1\}$ *with the property* $D(4k+3)$*.*

Proof. Let

$$2k+1=(2l+1)m,$$

where $l\notin\{-1,0\}$ and $m\geq 3$. Then $4k+3=2(2l+1)m+1$. Set

$$\begin{aligned} a &= m \\ b &= (3l+1)^2m+2l \\ c &= (3l+2)^2m+2l+2 \\ d &= 9(2l+1)^2m+8l+4. \end{aligned}$$

We claim that the set $\{a,b,c,d\}$ has the desired property. By [4, (13)] it suffices to show that a, b, c and d are distinct integers and $b,c,d\geq 2$. Since $l\notin\{-1,0\}$, we have:

$$\begin{aligned} b-a &= (9l^2+6l)m+2l\geq 27l^2+20l>0 \\ c-a &= (9l^2+12l+3)m+2l+2\geq 27l^2+28l+11>0 \\ d-a &= (36l^2+36l+8)m+8l+4\geq 144l^2+152l+36>0 \\ c-b &= 3(2l+1)m+2\neq 0 \end{aligned}$$

$$\begin{aligned} d-b &= (3l+2)[(9l+4)m+2] \neq 0 \\ d-c &= (3l+1)[(9l+5)m+2] \neq 0. \end{aligned}$$

Hence a, b, c and d are distinct integers and $b, c, d > a \geq 3$. □

Lemma 3. *If the integer $|2k-3|$ is composite and $k \notin \{-3, 6\}$, then there exists a Diophantine quadruple $\{a, b, c, d\} \subset \mathbf{N} \setminus \{1\}$ with the property $D(4k+3)$.*

Proof. Let

$$2k - 3 = (2l+1)m,$$

where $l \notin \{-2, -1, 0, 1\}$ and $m \geq 3$. Then $4k+3 = 2(2l+1)m + 9$. Set

$$\begin{aligned} a &= m \\ b &= l^2 m + 2l - 2 \\ c &= (l+1)^2 m + 2l + 4 \\ d &= (2l+1)^2 m + 8l + 4. \end{aligned}$$

To prove that the set $\{a, b, c, d\}$ has the desired property, by [4, (23)], it suffices to show that a, b, c and d are distinct integers and $b, c, d \geq 2$. Since $l \notin \{-2, -1, 0, 1\}$, we have:

$$\begin{aligned} b-a &= (l^2-1)m + 2l - 2 \geq 3l^2 + 2l - 5 > 0 \\ c-a &= (l^2+2l)m + 2l + 2 \geq 3l^2 + 8l + 2 > 0 \\ d-a &= (4l^2+4l)m + 8l + 4 \geq 12l^2 + 20l + 4 > 0 \\ c-b &= (2l+1)m + 6 \neq 0 \\ d-b &= (l+1)[(3l+1)m+6] \neq 0 \\ d-c &= l[(3l+2)m+6] \neq 0, \end{aligned}$$

which gives the desired conclusion. □

Lemma 4. *If the integer $|18k+13|$ is composite, then there exist a Diophantine quadruple $\{a, b, c, d\} \subset \mathbf{N} \setminus \{1\}$ with the property $D(4k+3)$.*

Proof. Let

$$18k + 13 = (2l+3)m, \tag{2.2}$$

where $l \notin \{-2, -1\}$ and $m \geq 5$. Then $4k+3 = [2m(2l+3)+1]/9$. Set

$$\begin{aligned} a &= m \\ b &= (l^2 m - 2l - 6)/9 \\ c &= [(l+3)^2 m - 2l]/9 \\ d &= [(2l+3)^2 m - 8l - 12]/9. \end{aligned}$$

The numbers b, c and d are integers, by (2.2). We claim that the set $\{a, b, c, d\}$ has the desired property. From [6, proof of Theorem 1] it follows that the product of any two distinct elements of this set is a perfect square. Thus it suffices to prove that a, b, c and d are distinct integers and $b, c, d \geq 2$. Since $l \not\equiv 0 \pmod 3$, we have:

$$\begin{aligned}
b-a &= (l+3)[(l-3)m-2]/9 \neq 0 \\
c-a &= l[(l+6)m-2]/9 \neq 0 \\
d-a &= [(4l^2+12k)m-8l-12]/9 \geq (20l^2+52l-12)/9 > 0 \\
c-b &= [(2l+1)m+2]/3 \neq 0 \\
d-b &= (l+1)[(l+3)m-2]/3 \neq 0 \\
d-c &= (l+2)[lm-2]/3 \neq 0.
\end{aligned}$$

It remains to prove that $b \geq 2$ and $c \geq 2$. Since $k \notin \{-3,-2\}$ and $m \geq 5$, we have:

$$c \geq \frac{1}{9}(5k^2+28k+45) > 1.$$

Suppose that $k \neq 1$. Then $b \geq (5k^2-2k-6)/9 > 1$. If $k=1$, then from $54q+49=5m$ if follows that $m \geq 53$ and $b=(m-8)/9 > 5$. □

Proof of Theorem 1. It is clear that the assertion is valid for $n=15$. If $n \neq 15$, then there exist at least two distinct Diophantine quadruples with the property $D(n)$ which contain the number 1 (see [4, (7) and (17)]).

If either of the integers $|n-1|/2$, $|n-9|/2$ and $|9n-1|/2$ is not prime, then it is composite. Note that the integer $|27\cdot 9-1|/2=121$ is composite. From Lemmas 2, 3 and 4 it follows that there exists a Diophantine quadruple with the property $D(n)$ which does not contain the number 1. This contradicts to our assumption.

Suppose that $|n|$ is not a prime and that n is not a product of twin primes. If $n \equiv 0 \pmod 3$ and $n \notin \{3,15\}$, the integer $|n-9|/2$ is composite.

Thus $n \not\equiv 0 \pmod 3$ and $n=st$, where $s \geq 5$, $|t| \geq 5$ and $s-t > 2$. Write $v=(s-t-2)/4$. Then the set (2.1) is the Diophantine quadruple with the property $D(n)$, by Lemma 1. We claim that this quadruple is different from quadruples [4, (7)] and [4, (17)]. Indeed, the sums of elements of quadruples (2.1), [4, (7)] and [4, (17)] are $3(18v^2+18v+5+4vt+2t)$, $3(18k^2+22k+7)$ and $3(2k^2-2k-1)$, respectively, where $n=4k+3$. Since $18k^2+22k+7 \geq 2k^2-2k-1$ for every integer k, it is sufficient to prove that

$$18v^2+18v+5+4vt+2t < 2k^2-2k-1. \tag{2.3}$$

The relation (2.3) is equivalent to

$$(2v+1)(2v+4s+1) < \frac{1}{4}(n-1)(n-9). \tag{2.4}$$

Let $t > 0$. Then

$$v \leq \frac{\frac{n}{5}-5-2}{4} = \frac{n-35}{20}$$

and

$$(2v+1)(2v+4s+1) \leq \frac{n-25}{10}\cdot\frac{9n-25}{10}.$$

If $t<0$, then $n=-m<0$, and we have

$$v \leq \frac{\frac{m}{5}+5-2}{4} = \frac{m+15}{20}$$

and

$$(2v+1)(2v+4s+1) \leq \frac{m+25}{10} \cdot \frac{9m+25}{10} = \frac{n-25}{10} \cdot \frac{9n-25}{10}.$$

If $|n|>5$, then $(n-25)(9n-25)<25(n-1)(n-9)$, which establishes the formula (2.4) and completes the proof. □

3. The case n=8k+5

Theorem 2. *Let n be an integer such that $n \equiv 5 \pmod 8$, $n \notin \{-27,-3,5,13,21,45\}$, and there exist at most two distinct Diophantine quadruples with the property $D(n)$. Then the integers $|n|$, $|n-1|/4$, $|n-9|/4$ and $|9n-1|/4$ are primes.*

To prove this theorem we need the following lemmas.

Lemma 5. *Let n be an integer such that $n \equiv 5 \pmod 8$ and let $n=st$, where s and t are integers such that $s \geq 1$, $s-t>4$ and $t \neq -3s$. If $v=(s-t-4)/8$, then the set*

$$\{2, 2(3v+1)^2+2vt, 2(3v+2)^2+2(v+1)t, 18(2v+1)^2+4(2v+1)t\} \qquad (3.5)$$

is the Diophantine quadruple with the property $D(n)$.

Proof. From $st \equiv 5 \pmod 8$ it follows that $s \equiv t+4 \pmod 8$. Hence v is a positive integer. Set

$$\begin{aligned} b &= 2(3v+1)^2+2vt \\ c &= 2(3v+2)^2+2(v+1)t \\ d &= 18(2v+1)^2+4(2v+1)t. \end{aligned}$$

By [6, proof of Theorem 1], it is sufficient to prove that 2, b, c and d are distinct positive integers. We have:

$$\begin{aligned} b-2 &= 2v(9v+t+6) = 2v(v+s+2) > 0 \\ c-2 &= 2(v+1)(9v+t+3) = 2(v+1)(v+s-1) > 0 \\ d-2 &= 2(2v+1)(18v+2t+9)-2 = 2[(2v+1)(2v+2s+1)-1] > 0 \\ c-b &= 2(6v+t+3) = (3s+t)/2 \neq 0 \\ d-b &= 2(3v+2)(9v+t+4) = 2(3v+2)(v+s) > 0 \\ d-c &= 2(3v+1)(9v+t+5) = 2(3v+1)(v+s+1) > 0, \end{aligned}$$

which proves the lemma. □

Lemma 6. *If the integer $|2k+1|$ is composite, then there exists a Diophantine quadruple $\{a,b,c,d\} \subset \mathbf{N} \setminus \{2\}$ with the property $D(8k+5)$.*

Proof. Let

$$2k+1 = (2l+1)m,$$

where $l \notin \{-1,0\}$ and $m \geq 3$. Then $8k+5 = 4(2l+1)m+1$. Set

$$\begin{aligned} a &= 2m \\ b &= 2m(3l+1)^2 + 2l \\ c &= 2m(3l+2)^2 + 2l + 2 \\ d &= 18m(2l+1)^2 + 8l + 4. \end{aligned}$$

An analysis similar to the one in the proof of Lemma 2 shows that the set $\{a,b,c,d\}$ has the desired property. □

Lemma 7. *If the integer $|2k-1|$ is composite and $k \notin \{-4,5\}$, then there exists a Diophantine quadruple $\{a,b,c,d\} \subset \mathbf{N} \setminus \{2\}$ with the property $D(8k+5)$.*

Proof. The proof of Lemma 7 is completely analogous to the proof of Lemma 3. □

Lemma 8. *If the integer $|18k+11|$ is composite and $k \notin \{-2,3\}$, then there exist a Diophantine quadruple $\{a,b,c,d\} \subset \mathbf{N} \setminus \{2\}$ with the property $D(8k+5)$.*

Proof. Let

$$18k+11 = (2l+3)m, \tag{3.6}$$

where $l \notin \{-2,-1\}$ and $m \geq 5$. Then $8k+5 = [4m(2l+3)+1]/9$. Set

$$\begin{aligned} a &= 2m \\ b &= (2ml^2 - 2l - 6)/9 \\ c &= [2m(l+3)^2 - 2l]/9 \\ d &= [2m(2l+3)^2 - 8l - 12]/9. \end{aligned}$$

We claim that the set $\{a,b,c,d\}$ has the desired property. Let us first observe that (3.6) implies that b, c and d are integers. Similarly, as in the proof of Lemma 4 we obtain that a, b, c and d are distinct integers and $d > a$.

If $l \neq 1$, then $b \geq (10l^2 - 2l - 6)/9 > 2$, and if $l = 1$, then from $18l + 11 = 5m$ and $k \neq 3$ it follows that $m \geq 31$ and $b = (2m-8)/9 \geq 6$.

If $l \neq -4$, then $c \geq (10l^2 + 58l + 90)/9 > 2$, and if $l = -4$ then from $18k+11 = -5m$ and $k \neq -2$ it follows that $m \geq 23$ and $c = (2m+8)/9 \geq 6$. □

Proof of Theorem 2. If n satisfies the assumptions of Theorem 2 then there exist at least two distinct Diophantine quadruples with the property $D(n)$ which contain the number 2 (see [4, (9) and (19)]).

If either of the integers $|n-1|/4$, $|n-9|/4$ and $|9n-1|/4$ is not prime, then it is composite. Assume that $n \notin \{-11, 29\}$. Then from Lemmas 6, 7 and 8 it follows that there exists a Diophantine quadruple with the property $D(n)$ which does not contain the number 2.

For $n = -11$ the construction of Lemma 8 gives the quadruple $\{2, 10, 18, 30\}$, while [4, (9) and (19)] gives the quadruples $\{2, 30, 46, 150\}$ and $\{2, 6, 10, 30\}$ with the property $D(-11)$.

For $n = 29$ the construction of Lemma 8 gives the quadruple $\{2, 26, 46, 70\}$, while [4, (9) and (19)] gives the quadruples $\{2, 206, 250, 910\}$ and $\{2, 10, 26, 70\}$ with the property $D(29)$. This completes the proof that the integers $|n-1|/4$, $|n-9|/4$ and $|9n-1|/4$ are primes.

It remains to prove that $|n|$ is prime. Suppose that the integer $|n|$ is composite. We need to consider three cases.

First, let $n \equiv 0 \pmod 3$. Since $n \notin \{-3, 21\}$, the integer $|n-9|/4$ is composite, a contradiction.

Next, let $n = st$, where $s \geq 5$, $|t| \geq 5$ and $s - t > 4$. Let $v = (s-t-4)/8$. Then the set (3.5) is the Diophantine quadruple with the property $D(n)$, by Lemma 5. We will show that this quadruple is different from quadruples [4, (9)] and [4, (19)]. Indeed, the sums of elements of quadruples (3.5), [4, (9)] and [4, (19)] are $6(18v^2 + 18v + 5 + 2vt + t)$, $6(18k^2 + 20k + 6)$ and $12k^2$, respectively, where $n = 8k + 5$. Thus, it is sufficient to prove that

$$18v^2 + 18v + 5 + 2vt + t < 2k^2, \tag{3.7}$$

or, equivalently, that

$$(2v+1)(2v+2s+1) < \frac{1}{16}(n-1)(n-9). \tag{3.8}$$

The proof of (3.8) is completely analogous to the proof of (2.4).

Finally, let $n = pq$, where p and q are primes and $p - q = 4$. Since $n \neq 21$, we conclude that n is of the form $n = (6x+1)(6x+5)$, $x \geq 1$. An easy computation shows that the set

$$\{2, 32x^2 + 32x + 10, 288x^4 + 672x^3 + 542x^2 + 178x + 22,$$
$$288x^4 + 480x^3 + 254x^2 + 42x + 2\}$$

is the Diophantine quadruple with the property $D((6x+1)(6x+5))$. From $32x^2 + 32x + 10 < n$ it follows easily that this quadruple is different from quadruples [4, (9)] and [4, (19)]. This completes the proof of the theorem. □

4. Connection with Dickson's conjecture

Let U denote the set of all integers n, not of the form $4k+2$, such that there exist at most two distinct Diophantine quadruples with the property $D(n)$. It is not yet known, whether the set U is finite or not. From the results of [8] and Theorems 1 and 2 it follows that if U is infinite then at least one of the sets

$$\begin{aligned} A &= \{k \in \mathbf{Z} : |2k-3|, |2k+1|, |4k+3|, |18k+13| \ \text{are primes}\}, \\ B &= \{l \in \mathbf{N} : 2l-1, 2l+1, 2l^2-5, 2l^2-1, 18l^2-5 \ \text{are primes}\}, \\ C &= \{k \in \mathbf{Z} : |2k-1|, |2k+1|, |8k+5|, |18k+11| \ \text{are primes}\} \end{aligned}$$

is infinite. The question whether the sets A, B and C are infinite or not is still unanswered. Let us observe that the linear polynomials appearing in the sets A and C satisfy the conditions of following Dickson's conjecture ([10, p. 292]):

Let $s \geq 1$, $f_i(x) = b_i x + a_i$, with a_i, b_i integers, $b_i \geq 1$ (for $i = 1, \dots, s$). Assume that the following condition is satisfied:

There does not exist any integer $n > 1$ dividing all the products $f_1(k) f_2(k) \cdots f_s(k)$ for every integer k.

Then there exist infinitely many natural numbers m such that all numbers $f_1(m), f_2(m), \dots, f_s(m)$ are primes.

Indeed, if $f_1(x) = 2x-3$, $f_2(x) = 2x+1$, $f_3(x) = 4x+3$ and $f_4(x) = 18x+13$, then the integers $f_1(0)f_2(0)f_3(0)f_4(0) = -117$ and $f_1(2)f_2(2)f_3(2)f_4(2) = 2695$ are relatively prime, and if $g_1(x) = 2x-1$, $g_2(x) = 2x+1$, $g_3(x) = 8x+5$ and $g_4(x) = 18x+11$, then the integers $g_1(0)g_2(0)g_3(0)g_4(0) = -55$ and $g_1(1)g_2(1)g_3(1)g_4(1) = 1131$ are relatively prime. Furthermore, the polynomials from the set B satisfy the conditions of the Schinzel-Sierpiński conjecture ([11], [10, p. 312]), which is an analogue of Dickson's conjecture for irreducible polynomials. Therefore, the validity of the Schinzel-Sierpiński conjecture would imply that the sets A, B and C are infinite.

References

[1] Baker, A., Davenport, H., The equations $3x^2 - 2 = y^2$ and $8x^2 - 7 = z^2$. Quart. J. Math. Oxford Ser. (2) 20 (1969), 129–137

[2] Brown, E., Sets in which $xy + k$ is always a square. Math. Comp. 45 (1985), 613–620

[3] Diophantus of Alexandria, Arithmetics and the Book of Polygonal Numbers. Nauka, Moscow 1974 (in Russian)

[4] Dujella, A., Generalization of a problem of Diophantus. Acta Arith. 65 (1993), 15–27

[5] —, Diophantine quadruples for squares of Fibonacci and Lucas numbers. Portugal. Math. 52 (1995), 305–318

[6] —, Some polynomial formulas for Diophantine quadruples. Grazer Math. Ber. (to appear)

[7] —, The problem of Diophantus and Davenport for Gaussian integers. Glas. Mat. Ser. III (to appear)

[8] —, Some estimates of the number of Diophantine quadruples. (preprint)

[9] Gupta, H., Singh, K., On k-triad sequences. Internat. J. Math. Math. Sci. 8 (1985), 799–804

[10] Ribenboim, P., The Book of Prime Number Records. Springer Verlag, New York-Berlin-Heidelberg 1989

[11] Schinzel, A., Sierpiński, W., Sur certaines hypothèses concernant les nombres premiers. Acta Arith. 4 (1958), 185–208, Corrigendum, 5 (1959), 259